9. Problem sheet for Set Theory, Winter 2012

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Problem 31. Show that the sets $\{f \mid f\colon \omega \to 2\}$, $\{f \mid f\colon \omega \to \omega,\ f \text{ bijective }\}$, $\{f \mid f\colon \omega \to \omega,\ f \text{ surjective }\}$, and $\{f \mid f\colon \mathbb{R} \to \mathbb{R},\ f \text{ continuous}\}$ have the same cardinality.

Problem 32. Suppose A is a set and κ is a cardinal. Define:

- $[A]^{\kappa} = \{X \subseteq A \mid card(X) = \kappa\}$,
- $[A]^{<\kappa} = \{X \subseteq A \mid card(X) < \kappa\}$, and
- $[A]^{\leq\kappa} = \{X \subseteq A \mid card(X) \leq \kappa\}$.

Show: if A and κ are infinite and $\kappa \leq card(A)$, then $card([A]^{\kappa}) = card([A]^{\leq\kappa}) = card(A)^{\kappa}$ and $card([A]^{<\kappa}) = \sup\{card(A)^{\lambda} \mid \lambda < \kappa,\ \lambda \in Card\}$.

Problem 33. Show $\kappa < \kappa^{cof(\kappa)}$ by disproving the existence of a surjection $f\colon \kappa \to \kappa^{cof(\kappa)}$ by diagonalization, without using König's Theorem.

Problem 34. Prove $\Pi_{n<\omega}\aleph_n = \aleph_{\omega}^{\aleph_0}$, $\Pi_{\alpha<\omega+\omega}\aleph_\alpha = \aleph_{\omega+\omega}^{\aleph_0}$, and $\Pi_{\alpha<\omega_1+\omega}\aleph_\alpha = \aleph_{\omega_1+\omega}^{\aleph_1}$.

There are 6 points for each problem. Please hand in your solutions on Monday, December 10 before the lecture.